Journal Club talk – Causality and Locality, etc...

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1 Introduction

Causality and Locality are important parts of our understanding of General relativity. Very loosely

- Locality : Data about how a physical system will change is stored locally, i.e. the only data that can be relevant for the near-future dynamics of a system is data near that point in spacetime.
- Causality : Information cannot propogate on spacetime faster than the speed of light in a vacuum, c (strictly speaking this should be called the speed of causality). [1]

Both of these ideas are very important to our understanding of classical theories of the universe, but somehow everyone is cool with the fact that the first one is just false (probably because it turns out that the second stops you from doing most of the bad you could do with the first). I'm going to try to touch on what a few current views of physics show us about causality and locality, and what this has to do wit the research I do in neutron stars.[2]

2 General Relativity: A failing theory that doesn't fail

As far as we can tell every prediction of Einstein's theory of General Relativity is correct. Some examples

- Deflection of light by sun 1973 result Found $.95 \pm .11$ times the GR value (i.e. inconsistent with the Newtonian value 1/2 the GR value)
- Perihelion Precession of Mercury 575.31 (theoretical) vs 574.10 (experimental)
- Gravitational Waves – 150914 happened at about 5.1 σ level, precision tests of GR are difficult but gravitational radiation consistent with GR at Advanced LIGO sensitivity is basically guaranteed to be present in the correct theory of gravity.

That being said, GR has written into it the makings of it's own demise.

[1] In QFT causality is stronger than this, in particular it says that $[\phi(x_1), \phi(x_2)] = 0$ if x_1 is not on the light cone of x_2 (same statement can be read with x_1 and x_2 flipped) if ϕ is a massless field.

[2] A lot of this talk is based on lectures given by Tom Hartman at Cornell, I'm not sure if a public repo of materials exist but there are published materials which contain even more info than I'm presenting here. See for example Tom Hartman's page on a recent BH information course

2.1 When is GR Effective?

Every indication says that GR is an *effective theory*, i.e. not necessarily fundamental, but useful in it's domain of applicability. General relativity is needed to describe gravity when

$$\frac{GM}{rc^2} \not\ll 1 \tag{1}$$

Notice that, contrary to popular belief, General relativity is not necessarily important when density is large, but rather it is much more strongly a function of when mass is large. Therefore even very diffuse galaxy clusters can bend light, while extremely dense atomic nuclei cannot.

We use the Schwarzschild spacetime as a prototype for a massive object in GR, even though the Kerr metric may be more appropriate in many situations (in units where $c = G = 1$).

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2 \tag{2}$$

This Taylor expands to, in the case of weak gravity,

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 + \frac{2M}{r}\right) dr^2 + r^2 d\Omega_2^2 \tag{3}$$

For a massive particle $-ds^2 = d\tau^2$, we can write for a radially falling object

$$-1 = -\left(1 - \frac{2M}{r}\right)\left(\frac{dt}{d\tau}\right)^2 + \left(1 + \frac{2M}{r}\right)\left(\frac{dr}{d\tau}\right)^2 \tag{4}$$

For slowly moving observers, $\frac{dr}{d\tau}$ is small compared to $\frac{dt}{d\tau}$, in this case, to recover fully Newtonian theory ignore higher order terms in $\frac{dr}{d\tau}$ and $\frac{2M}{r}$ (or any combo), and set $\frac{dt}{d\tau} = 1$.

$$\frac{2M}{r} = \left(\frac{dr}{dt}\right)^2, \tag{5}$$

So Newtonian gravity is an effective theory for GR in the low-mass, low-velocity limit.

This approximation breaks down of course when $\frac{2M}{r}$ is not small though, and doubly so when $\frac{dr}{d\tau}$ is not small, in that case we have no choice but to use the full theory of GR. This is very natural, we do not expect Newtons gravity to work when. M/r is getting large, especially as it diverges.

GR also has some problems though, for example the metric coefficients in the Schwarzschild spacetime go wacko as $2M/r \to 1$. Surprisingly though this is not a problem, as Einstein didn't immediately appreciate, this is just a coordinate feature of GR, and doesn't signal the breakdown of the theory. There is something more though, the fact that the Schwarzschild coordinates break down at the event horizon signals there's some problem in connecting the horizon to infinity. In particular a clock ticking at the horizon never advances from the point of view of an observer at spatial infinity. This is a nonlocal detail, locally everything is fine (if you can avoid being ripped apart by tidal forces, as you fall through the horizon along with the clock it will continue to click as usual). It appears that the horizon is a true feature of GR, and doesn't in any way signal a breakdown of the theory, rather it is indicative of the fact that there are nonlocal features of GR that don't have any local signature – The only way to know if you're inside an event horizon is to wait until future infinity and find that you still haven't been able to make it to null infinity by moving at the speed of light.

There is an actual problem though. Despite the Schwarzschild spacetime being a vacuum spacetime and having $R = 0$, $R_{\mu\nu} = 0$ everywhere, the curvature invariant, called the Kretschmann scalar,

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48\frac{M^2}{r^6} \tag{6}$$

actually diverges as $r \to 0$ at constant M. This isn't usually a problem because the quantity $\frac{M}{r^3}$ is usually constant at the cores of massive objects, as it approaches something proportional to the central density ρ_c. This indicates that GR will break down at the center of a Schwarzschild black hole if it hasn't already broken down by then.

Note:
GR breaking down is really a *local* property, because it is based on the density which is a local property. This is satisfying, as GR is a local theory (the field equations are local PDEs) GR being important is not really about local details but about large scale details, since it depends more strongly on the total mass in some region, rather than the mass at a point. This is fine though, as Newtonian gravity is inherently nonlocal, so we end up exactly where we expect.

2.2 Practical implications: AKA Black Holes are not necessarily dense

We can approximate what the value of the Kretschmann scalar should be in the center of a NS, since we actually know to some degree what density means there, generally ρ_c (note this is the energy density in a relativistic sense) should clock in somewhere around $.003 - .01\frac{1}{M_\odot^2}$, this corresponds to length scales of something like $1.477\text{km}/M_\odot\sqrt{400/(4\pi/3)} \approx 15\text{km}$ or a bit smaller. This is in some sense the "radius" of curvature associated with GR effects at the center of a NS (it's also coincidentally about the radius of a NS). Some BH's could be denser than this (in the sense that the Kretschmann scalar could be larger at places we can probe), but, for example, the average "density" of a Schwarzschild BH is easy to compute

$$\bar{\rho} = \frac{M}{\frac{4\pi}{3}(2M)^3} \approx \frac{1}{32M^2} \tag{7}$$

If the lightest BH is say $5M_\odot$ then it is the densest BH and has average density $\bar{\rho} \approx \frac{1}{800}$, obviously though if there are $2.2M_\odot$ BHs then there may be places with comparable Krestchmann values (especially if the Nuclear EoS is softer at higher densities). We don't know at what length Scale GR will break down (there's perhaps no good reason to expect it to happen before the Planck length), but it's possible it happens at some non-tiny length scale. In which case, NSs may actually be better targets to probe for violations of GR than most BHs.

2.3 What might such a violation look like?

GR is an effective theory, which means the way we get it is by writing down an action consistent with some underlying principles. In this case that is coordinate and diffeomorphism invariance (some may call these the same thing but they really are different in this context). What this means is that the action must be composed of tensors with all indices contracted away, and can't change under a diffeomorphism $g_{\mu\nu} \to g_{\mu\nu} + \nabla_{[\nu}\xi_{\mu]}$. In practice this means the action has to be an expansion in curvature; the action of GR is called the Einstein-Hilbert Action.

$$S = \int \frac{1}{2\kappa} (2\Lambda + R)\sqrt{-g} d^4x \tag{8}$$

Where Λ is the constant term in some taylor series and R the Ricci scalar curvature is the leading order term, and κ is the constant that gets the theory to agree with Newton at low densities. The Ricci scalar is the only curvature term which meets our requirements and is first order. For some reason Λ is really small, (much less than what we would expect for a reasonable Taylor series of an arbitrary function), but we won't get into that now. Varying this action gives the Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{9}$$

We expect that there very well could be quadratic corrections to the action

$$S = \int \frac{1}{2\kappa} (2\Lambda + R) + c_1 R^2 + c_2 R_{\alpha\beta} R^{\alpha\beta} + c_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + ...\sqrt{-g} d^4x \tag{10}$$

The constants c_1, c_2, c_3 must have dimensions of length^2 = energy^2 to make all the terms have the same dimensions in the action. Therefore these terms will become important when, for example $c_3|R^{\alpha\beta\gamma\delta}| \approx 1$. This is similar to what we said before, but more concrete, if the length scales associated with new physics are large enough, then we will see them at smaller values of $|R^{\alpha\beta\gamma\delta}| \propto \rho$. We can compute the effects of such modifications to GR (sometimes), I think Masha Okounkova has done some work on this and would certainly know more.

3 Quantum Mechanics : Out of Left Field

I'm now going to pivot and talk about Quantum mechanics for no obvious reason other than the title of the talk. In quantum mechanics the idea of locality as an essential physical principle disappears, the reason is the Bell inequalities.

3.1 Bell Inequalities

In 1935 Einstein, Podolosky, and Rosen wrote a paper *Can Quantum-mechanical description of reality be considered complete?*. The authors conclusion was it could not, as correlation experiments in QM demonstrated there was information which QM couldn't predict but which according to a local theory of the universe should exist. Bell in a paper *On the Einstein*

Podolsky Rosen Paradox demonstrated that quantum information was fundamentally inconsistent with the concept of locality, so rather than quantum mechanics being incomplete, it was the classical ideal of local information determining physics which was incomplete.

The argument is remarkably simple for having been missed for 40 years (although maybe it's so banal nobody cared to note it before).

If Alice, and Bob set up a usual two particle experiment with entangled electrons, say.

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B \right) \tag{11}$$

Where A indicates Alice will measure the electron and B indicates Bob will measure the electron. If both Alice and Bob measure along the same axis, then there is perfect anticorrelation between Alice and Bob, $P(A = +1|B = -1) = 1$. We can say they *agree* on what happened every single time If Bob screws up and bumps his detector, by an angle $\delta\theta$, then the correlation changes. The will still be anticorrelated most of the time but $1 - \cos(\delta\theta) \approx \delta\theta^2/2$ they will actually come up with the same result. If, one day, Charlie noticed that Bob and Alice experiments are screwed up, he can note the degree of disagreement, and upon blaming Bob, inserts his own detector in place of Bob's but changing the angle by $\delta\theta$. The surprise is that Charlie corrected the wrong way, and his detector is now located an angle $2\delta\theta$ away from Alices. Einstein, seeing the mess they have created, makes an observation: a complete description of the hidden variables of the system would show that it doesn't matter if Charlie or Bob detects the particle opposite Alice's because if they are drawn from a constant ensemble governed by some hidden parameters λ. Charlie and Bob's detectors are not oriented that differently, so they would have agreed basically all the time if both of them could have made the measurement (but we assume each particle can only be measured once, otherwise it is contaminated by the previous measurement and we can't say anything about it). Likewise Alice and Bob's detectors will agree basically all the time. Bell points out, though, that Charlie will disagree with Alice $1-\cos(2\delta\theta) \approx 4\delta\theta^2/2 = 2\delta\theta^2$, and disagree with what Bob would have measured $\delta\theta^2/2$. Therefore if Bob disagrees with Alice 1/1000 measurements, then Charlie must disagree with Alice 4/1000 measurements. If the particles have hidden variables that are being drawn from some distribution, we can say that Bob and Charlie would have measured the same thing all but 1/1000 measurements, but if Bob and Alice agree, and Bob and Charlie would have agreed, then Charlie would have agreed with Alice. Therefore Charlie and Alice can only disagree if Bob disagrees with Alice or Charlie disagrees with Bob, which only happens at most 2/1000 measurements, only half as much as much as observed in QM experiments. The only conclusion that can reasonably be drawn is that Bob not having chose to make a measuremnt somehow changes the distribution of hidden variables (or vice versa) (JK lol that's superdeterminism). In fact the general solution is to abandon the concept of local hidden variables altogether, and with it the principle that physical laws should be local.

3.2 Causality

Remarkably to this day no such contradiction exists for the similarly Einsteinian notion that the laws of physics should be causal. It appears that Quantum information behaves in just such a way that it is not possible to transfer information faster than the speed of light. One way to see this generically is to use Density matrices. These objects quantify situations when there is uncertainty about which quantum state a particle is in. Note this is different from "quantum" uncertainty which is a feature solely of quantum mechanics, this is literally a probability distribution on quantum states. In general it is written like

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| \tag{12}$$

The type of sum we're doing is in an abstract vector space of operators, the reason we define it is because it is useful, this sum doesn't really mean anything a priori. If we are certain as to the quantum state, then only one state contributes to the sum

$$\rho = |\psi\rangle \langle\psi| \tag{13}$$

and the system is called a pure state. In this case there is no uncertainty as to the state of the system. That being said, individual observers are still uncertain about quantum information in the system. In the example above, Bob may want to know the probability he measures spin up or spin down. It turns out there is a very convenient way to compute this with this formalism using what's called a partial trace. In effect, Bob can *marginalize* over all of the possible results Alice can get (I.e. $\uparrow$ or $\downarrow$). It looks like this

$$\rho_B \equiv \langle\uparrow|_A \rho |\uparrow\rangle_A + \langle\downarrow|_A \rho |\downarrow\rangle_A \tag{14}$$

And here

$$\rho = |\psi\rangle \langle\psi| = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B) \frac{1}{\sqrt{2}} (\langle\uparrow|_A \langle\downarrow|_B + \langle\downarrow|_A \langle\uparrow|_B) \tag{15}$$

Alice's kets and bras don't interact with Bob's, so after some arithmetic this is

$$\rho_B = \frac{1}{2} |\uparrow\rangle_B \langle\uparrow|_B + \frac{1}{2} |\downarrow\rangle_B \langle\downarrow|_B \tag{16}$$

This is another density matrix, and it says that from Bob's point of view, being unable to interact with Alice's particle, it looks like he has a classical uncertainty, a coin flip chance of getting either $\uparrow$ or $\downarrow$. This is really what his odds are, but the point here is that even though the input state was pure, and had no classical uncertainty or *entropy*, the state when Bob only got to see his half did appear to have some entropy.

4 ER = EPR: do we know what we are approximating?

What happens when a black hole forms is somewhat uncertain; in principle with GRMHD simulations we can track the motion of matter through the event horizon but running such simulations for long periods of time with high resolution to yield good accuracy incorporating all known physics is still a pipe dream. Nonetheless when a BH does form, the No-hair theorem implies that eventually it settles down to a stationary state described by at most 3 numbers, mass, spin, and charge. This being said, QFT calculations in curved spacetime imply that a BH has many states, essentially proportional to the surface area of the BH and with a scaling of something like 1/plank areas. What gives? Is it possible to take a limit of the quantum system and get the classical result? Certainly doesn't seem like it as the Planck area is proportional to $\hbar$, so sending that sucker to zero will only make the situation worse. It turns out that the solution may be that the classical GR solutions we write down may describe both halves of, for example, the Schwarzschild spacetime, and that the solution we can observe and measure is only half of the quantum system. If this is the case, then the entropy we can observe may be quite a bit larger, (just like Bob observing more entropy in the parts than in the whole). I think someone has done calculations with the whole spacetime and showed the the total entropy of the Schwarzschild spacetime is indeed small when both halves of the system are included, even though the other even horizon is not causally connected to our universe. In principle, if Alice and Bob lived in separate black holes, then these black holes would be collecting correlated quantum information as the particles arrived. It is then not strictly possible in GR for Alice and Bob to communicate the results of their measurements to one another or even someone standing at the source of the particles somewhere between the two black holes. This may seem far fetched but surprisingly it should be something we are allowed to test without getting to the Planck scale, as it only depends on physics at low energies.

4.1 Causality as fundamental

Why locality has to be abandoned to move forward with writing physical laws but causality gets to stay is unclear, but probably for the best: physics doesn't really make sense in an acausal universe. That being said, causality is a fundamental feature of relativity; if the theory does break down it is not guaranteed that this will still feature in the effective field theory of gravity. The thing that is really weird in my opinion is that as far as we can tell nothing about our current theories of physics are breaking down at the event horizon of a black hole (including quantum field theory).