California Institute of Technology
Physics 101 Recitation
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What's Shaking?
1 May 2023

"We want a story that starts out with an earthquake and works its way up to a climax." —Samuel Goldwyn

Contents

1 Practice Problems

1.1 Birthday quake

(a) *How long does it take an earthquake on one side of the Earth to be detected on the other?*

Compare to Figure 1.

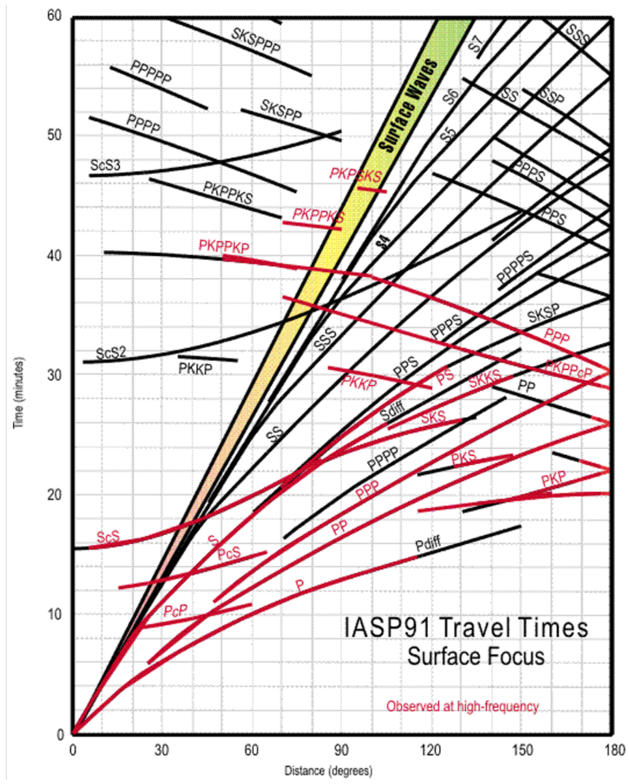

Figure 1: Earthquake travel times, from the United States Geological Survey.

Earthquakes are sound waves (either pressure- or shear-restored) which travel on the surface or interior of the Earth. Since the straightest path through the Earth is the interior, we guess that these sound waves will arrive at the other end of the Earth first.

The first step is then to estimate the sound speed in the Earth's interior. One way to do this is to write

$$c_s \sim \sqrt{\frac{P}{\rho}} \tag{S1}$$

where P and ρ are the characteristic pressures and densities in the Earth's interior. We can estimate P as the bulk modulus $\mathcal{B} \sim 10^{12}\,\mathrm{dyn\,cm^{-2}}$ (the upper value given by Purcell for elastic moduli), since the interpretation of the bulk modulus is the pressure needed to change the volume of a material by an e-folding:

$$\mathcal{B} \equiv -\frac{\mathrm{d}P}{\mathrm{d}\ln V} \tag{S2}$$

Guessing $\rho \simeq 3\,\mathrm{g\,cm^{-3}}$ (the density of rock), this would yield

$$c_s \simeq 6\,\mathrm{km\,s^{-1}} \tag{S3}$$

We could also approach this a different way, by noting that there is only one dimensionally unique way to construct a speed from G, $M_\oplus$, and $R_\oplus$:

$$c_s \simeq \sqrt{\frac{GM_\oplus}{R_\oplus}} \simeq 8\,\mathrm{km\,s^{-1}} \tag{S4}$$

These values roughly agree. Notably, in this second approach, $P \simeq GM^2/R^4 \simeq 10^{13}\,\mathrm{dyn\,cm^{-2}}$ and $\rho \simeq M_\oplus/4R_\oplus \simeq 6\,\mathrm{g\,cm^{-3}}$ (about twice as dense as everyday rock). The factor of 4 may translate to a factor of 2 difference in the sound speed.

The crossing time of a sound wave is then

$$t_{\mathrm{cross}} \simeq \frac{2R_\oplus}{c_s} \simeq 30\,\mathrm{min} \tag{S5}$$

This appears to agree to order of magnitude quite well with Figure 1.

(b) *How long does it take a starquake to cross the Sun?*

We see that the analysis in part (a) extends straightforwardly to the Sun. Notably, we guess

$$c_s \simeq \sqrt{\frac{GM_\odot}{R_\odot}} \simeq 400\,\mathrm{km\,s^{-1}} \tag{S6}$$

Then

$$t_{\text{cross}} \simeq \frac{2R_\odot}{c_s} \simeq 50\,\mathrm{min} \tag{S7}$$

In fact, aside from the factor of two, we actually could just have guessed t_{cross} on its own from dimensional grounds alone, since the above combination is the only dimensionally unique way to construct a time.

(c) *What is the frequency spacing between adjacent pressure modes in the Sun?*

By thinking about the normal modes in the Sun as oscillations in an infinite square well, we see that the frequency spacing ought to go as the inverse crossing time, i.e.,

$$\Delta\nu \simeq t_{\text{cross}}^{-1} \simeq 300\,\mu\mathrm{Hz} \tag{S8}$$

In reality, this so-called "large frequency spacing" is $\Delta\nu \approx 136\,\mu$Hz.

(d) *When dribbled, at what frequency does a basketball ring?*

A standard basketball has a radius $R \approx 12\,\mathrm{cm}$, and are filled with air (which has a speed of sound at standard temperatures and pressures $c_s \approx 343\,\mathrm{m\,s^{-1}}$).
Then

$$\Delta\nu \simeq \frac{c_s}{2R} \simeq 1.4\,\mathrm{kHz} \tag{S9}$$

which should also be close to the fundamental mode.

According to *Russell*[a], this number is pretty close to the frequency spacing between modes of adjacent radial order, and is also on the same order as the lowest radial-order modes.

[a] https://www.acs.psu.edu/drussell/Publications/Basketball.pdf

1.2 What's on your plate?

(a) *How much energy is dissipated in tectonic plate motions per year?*

Tectonic plates move by $\simeq 10$–$160\,\mathrm{mm\,yr^{-1}}$—we this assume $v \sim 0.6\,\mathrm{m\,yr^{-1}}$.

If pressures are characteristically $P \sim \mathcal{B} \simeq 10^{12}\,\mathrm{dyn\,cm^{-2}}$, then the normal force N between plates is

$$N \simeq \mathcal{B}A \simeq \mathcal{B}(\alpha R)h \tag{S10}$$

where the contact area between the plates scales with the plate thickness h (where $h \sim 10^2\,\mathrm{km}$ for oceanic plates, according to *The Geological Society of London*), and the total length of all crustal borders $\alpha R_\oplus$, where we guess $\alpha \simeq 10$.

Then the frictional force between the plates is

$$F \sim \mu N \tag{S11}$$

where we can guess $\mu \simeq 1$ for a very rough and interlocking rocky interface. Then the power dissipated is

$$P \simeq \alpha\mu\mathcal{B}R_\oplus hv \sim 10\,\mathrm{PW} \tag{S12}$$

It appears to still be an open question how much of this slip is *seismic* (associated with earthquakes), or *aseismic* ("creep" not associated with earthquakes), see, e.g., the 2003 textbook by *Stein* and *Wysession*.

(b) The Gutenberg–Richter law estimates that the number of earthquakes $N(M)$ with magnitude M will follow

$$N(M) \propto 10^{-bM} \tag{1}$$

If b is very low, the earthquake distribution is shallow, and the largest earthquakes should dominate the total energy in earthquakes. However, if b is very large, there are so few high-magnitude earthquakes that the low-magnitude earthquakes should dominate.

Use the fact that the energy in an earthquake scales with the Richter magnitude M as $10^{(3/2)M}$. *Find the parameter b such that the total energy contributed by earthquakes of all magnitudes is comparable.*

The total power in earthquakes $P(M)$ as a function of M is given by

$$P(M) \propto 10^{(3/2)M}N(M) = 10^{(3/2-b)M} \tag{S13}$$

We see that the threshold value of b is $b = 3/2$.

Notably, $b \approx 1$ typically, and so large earthquakes dominate the dissipated power of all earthquakes.