gravity waves in strong magnetic fields

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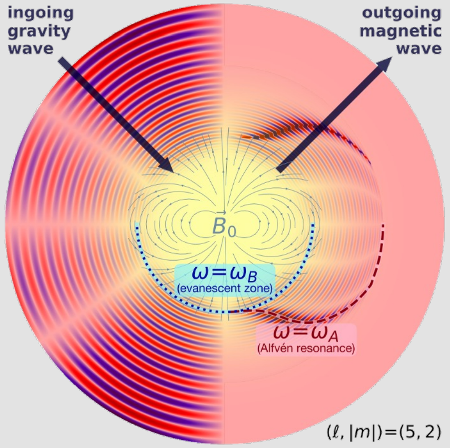
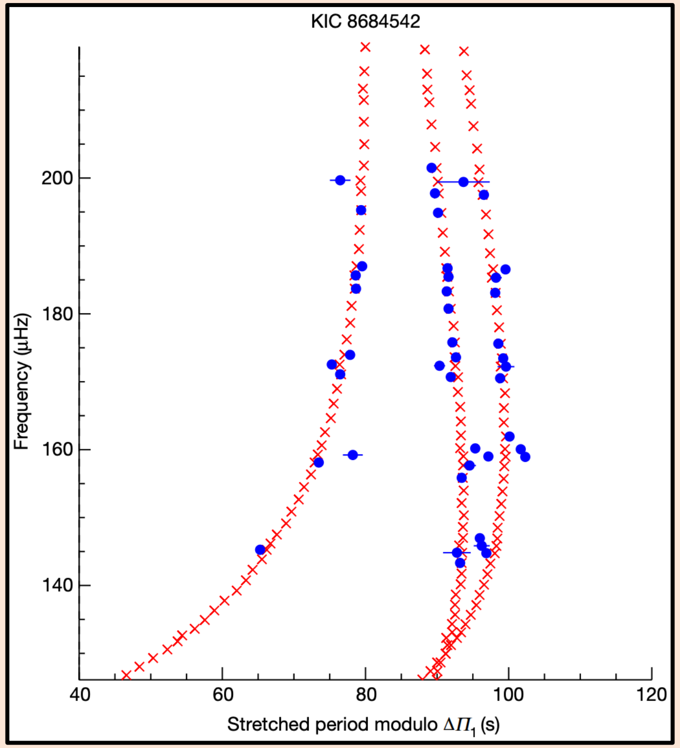
TASC7/KASC14 Workshop



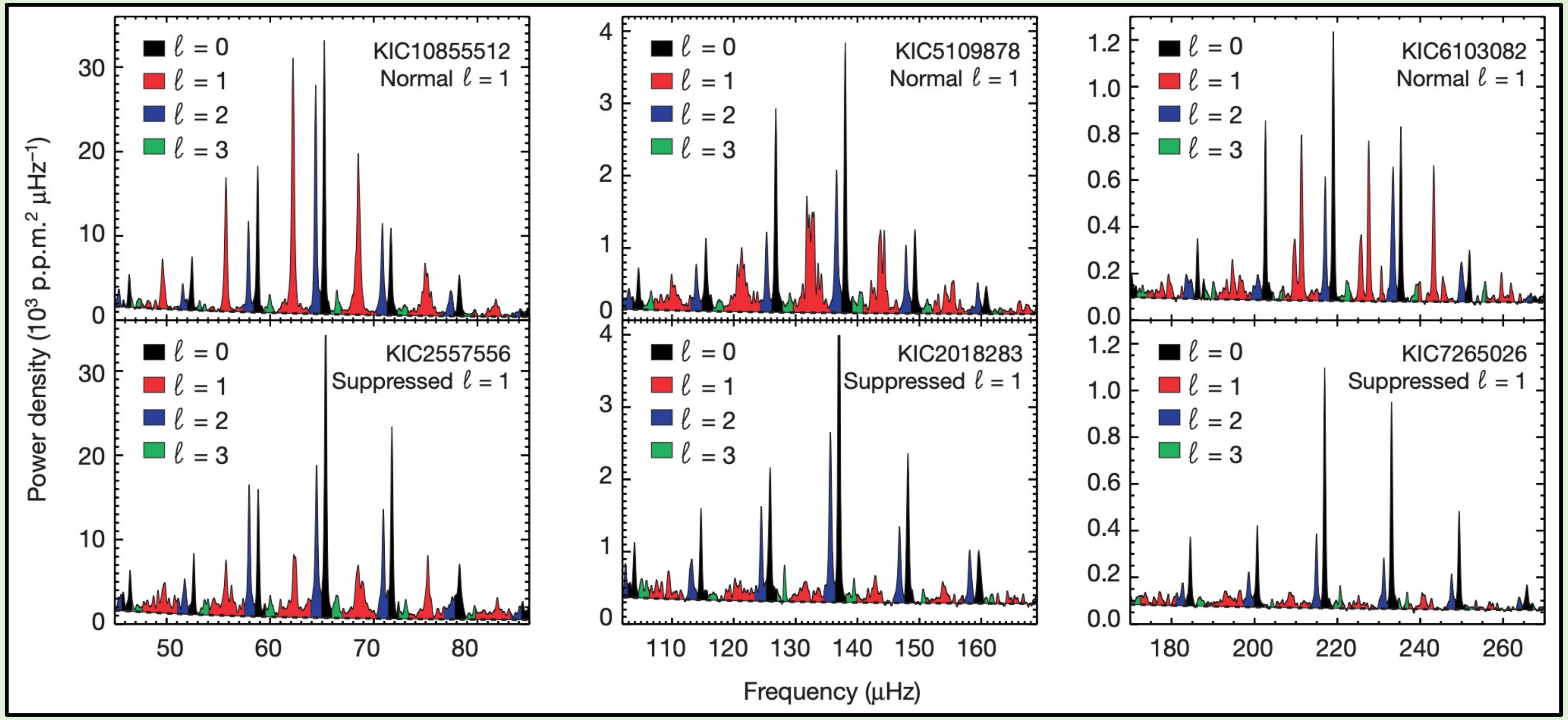
Caltech

stellar oscillations under magnetism

strong magnetism in stellar interiors can modify or even potentially suppress oscillations



Li, G., Deheuvels, S., Ballot, J. et al. *Nature* **610**, 43–46 (2022).



Stello, D., Cantiello, M., Fuller, J. et al. *Nature* **529**, 364–367 (2016).

strong magnetogravity waves

buoyancy is important:

$$k_r/k_h \sim N/\omega \gg 1$$

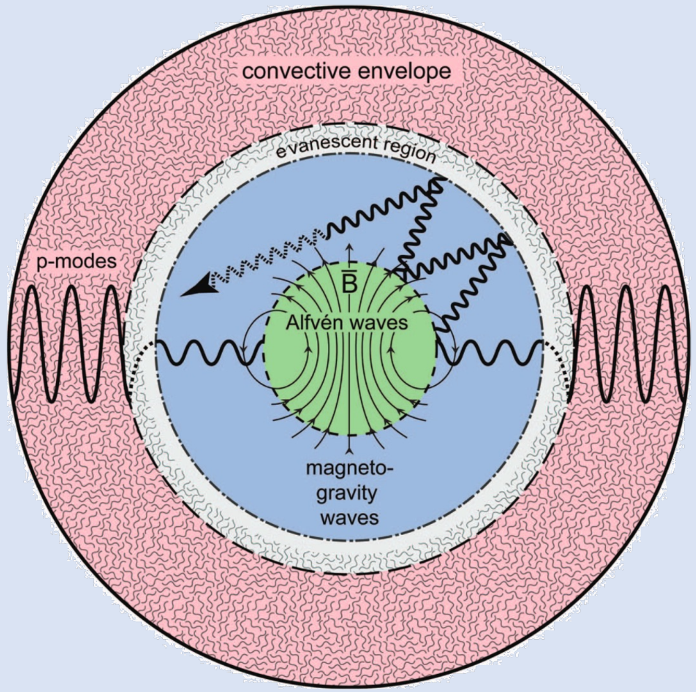
magnetism is important:

$$\omega_A \propto \mathbf{k} \cdot \mathbf{B} = k_r B_r + k_h B_h$$

∴ the radial field B_r is most impactful

Fuller et al. (2015): sufficiently strong fields prevent gravity waves from even propagating

$$\omega_B \simeq \sqrt{2 k_h v_{Ar} N}$$

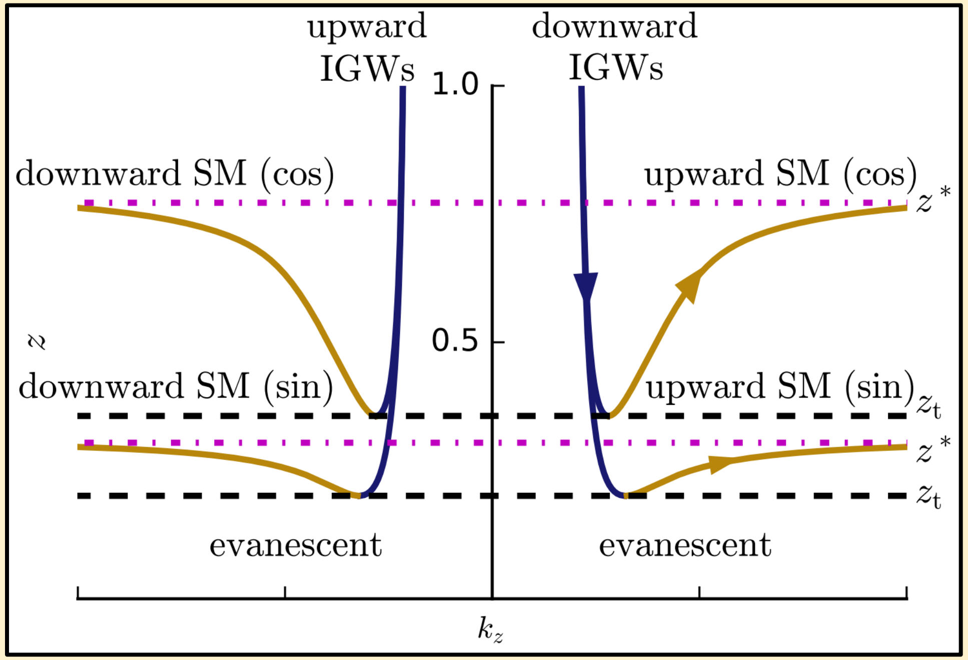


Fuller, J. et al. *Science* **350**, 423-426 (2015).

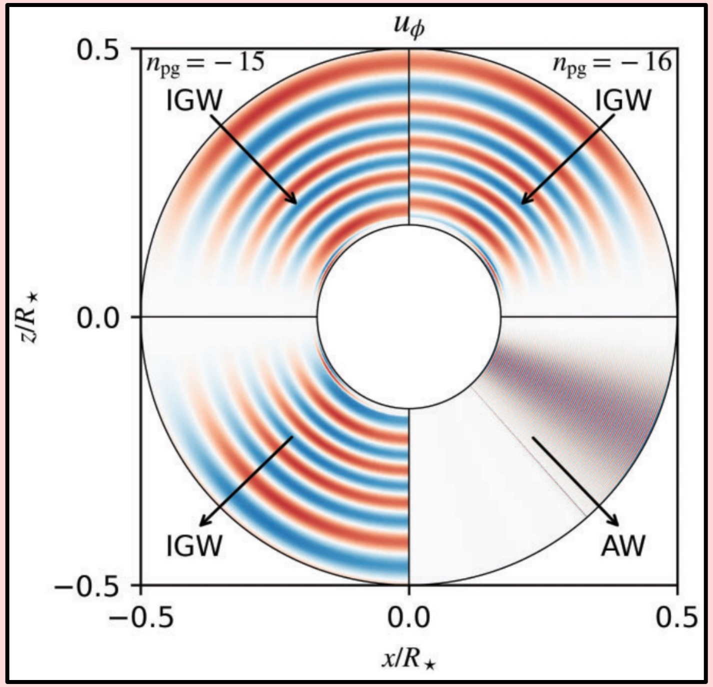
strong magnetogravity waves

Lecoanet et al. (2017):

in realistic geometries, strong magnetogravity waves up-refract to infinite wavenumber



Lecoanet, D. et al. *MNRAS* **466**, 2 (2017)



Lecoanet, D. et al. *MNRASL* **512**, 1 (2022).

Lecoanet et al. (2022):

tesseral/sectoral (m ≠ 0) modes have sharp fluid features near resonances with Alfvén waves

traditional approximation of rotation

solve perturbed fluid equations with new force:

$$\mathbf{f}_{\rm Coriolis} = 2\rho\dot{\xi} \times \mathbf{\Omega} \propto \omega\xi \times \mathbf{\Omega}$$

full-WKB dispersion relation:

$$\omega^2 - \frac{k_h^2}{|\mathbf{k}|^2} N^2 - (\hat{\mathbf{k}} \cdot \mathbf{\Omega})^2 = 0$$

buoyancy implies wavenumbers are mostly radial:

$$k_r / k_h \sim N/\omega \gg 1$$

only the radial part matters:

$$\mathbf{k} \cdot \mathbf{\Omega} = k_r \Omega_r + k_h \Omega_h \approx k_r \Omega_r$$

traditional approximation of rotation

relax the horizontal WKB approximation, but discard Ω_h:

$$\omega = \pm \frac{\tilde{k}_h}{k_r} N \text{ where } \tilde{k}_h = \sqrt{\lambda}/r$$

where Hough functions replace the spherical harmonics:

$$0 = \lambda p' + \frac{\mathrm{d}}{\mathrm{d}\mu}\left(\frac{1-\mu^2}{1-q^2\mu^2}\frac{\mathrm{d}p'}{\mathrm{d}\mu}\right) - \frac{m^2}{(1-\mu^2)(1-q^2\mu^2)}p' - \frac{mq(1+q^2\mu^2)}{(1-q^2\mu^2)^2}p'$$

$$q = 2\Omega/\omega$$
$$\mu = \cos\theta$$

inspiration for the magnetic problem?

“traditional approximation of magnetism”

solve perturbed fluid equations with new force:

$$\mathbf{f}_{\text{tension}} = (\mathbf{B} \cdot \nabla)\mathbf{B}'/4\pi \propto (\mathbf{k} \cdot \mathbf{B})^2 \xi$$

full-WKB dispersion relation:

$$\omega^2 - \frac{k_h^2}{\mathbf{k}^2} N^2 - (\mathbf{k} \cdot \mathbf{v}_A)^2 = 0 \qquad \left(\mathbf{v}_A = \mathbf{B}/\sqrt{4\pi\rho} \right)$$

buoyancy implies wavenumbers are mostly radial:

$$k_r/k_h \sim N/\omega \gg 1$$

only the radial part matters:

$$\mathbf{k} \cdot \mathbf{B} = k_r B_r + k_h B_h \approx k_r B_r$$

"traditional approximation of magnetism"

relax the horizontal WKB approximation, but discard B_h:

$$\omega = \pm \frac{\tilde{k}_h}{k_r} N \text{ where } \tilde{k}_h = \sqrt{\lambda}/r$$

where, for a dipole field,

$$0 = \lambda p' + \frac{\mathrm{d}}{\mathrm{d}\mu}\left(\frac{1-\mu^2}{1-b^2\mu^2}\frac{\mathrm{d}p'}{\mathrm{d}\mu}\right) - \frac{m^2}{(1-\mu^2)(1-b^2\mu^2)}p'$$

$$b = k_r v_{Ar}/\omega \sim \omega_A/\omega$$
$$\mu = \cos\theta$$

“traditional approximation of magnetism”

to avoid computing $b \propto k_r$ directly, rewrite dispersion relation as

$$b^2 = \lambda a^2$$

where

$$b = k_r v_{Ar}/\omega \sim \omega_A/\omega$$

$$a = \left(\frac{N}{\omega}\right)\left(\frac{v_{Ar}/r}{\omega}\right) \sim \omega_B^2/\omega^2$$

ω vs. ω_A : controls “structure” of modified spherical harmonic

ω_B : computable from stellar profile

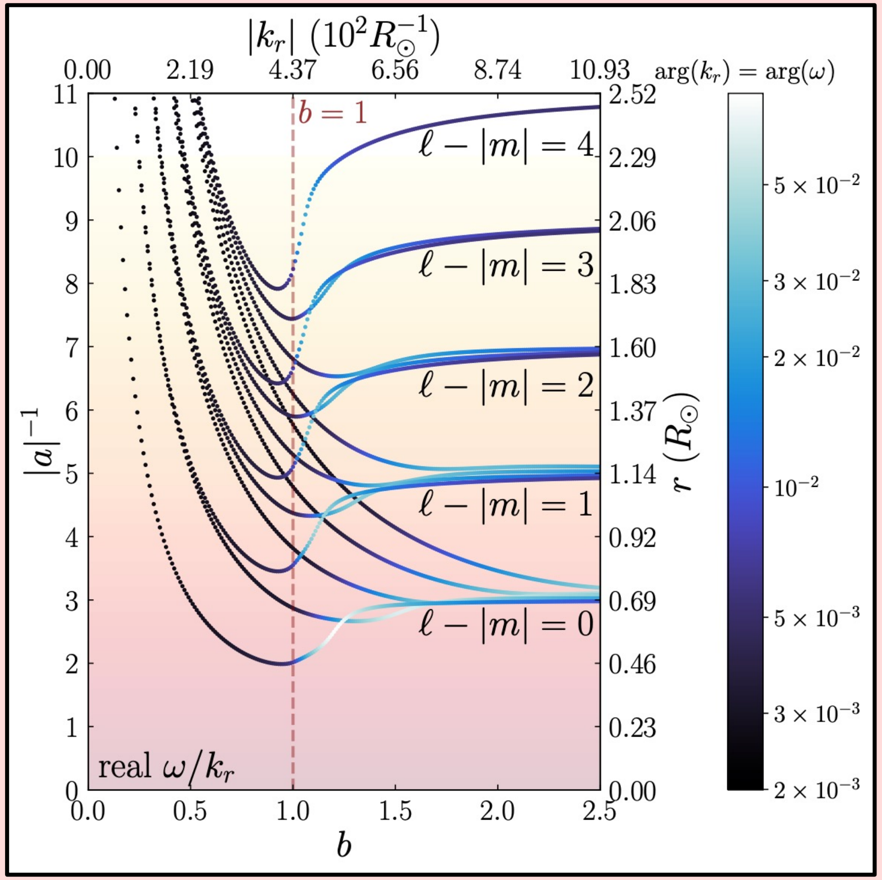
ω_A vs. ω_B : fixed by dispersion relation

side-by-side comparison: rotation and magnetism

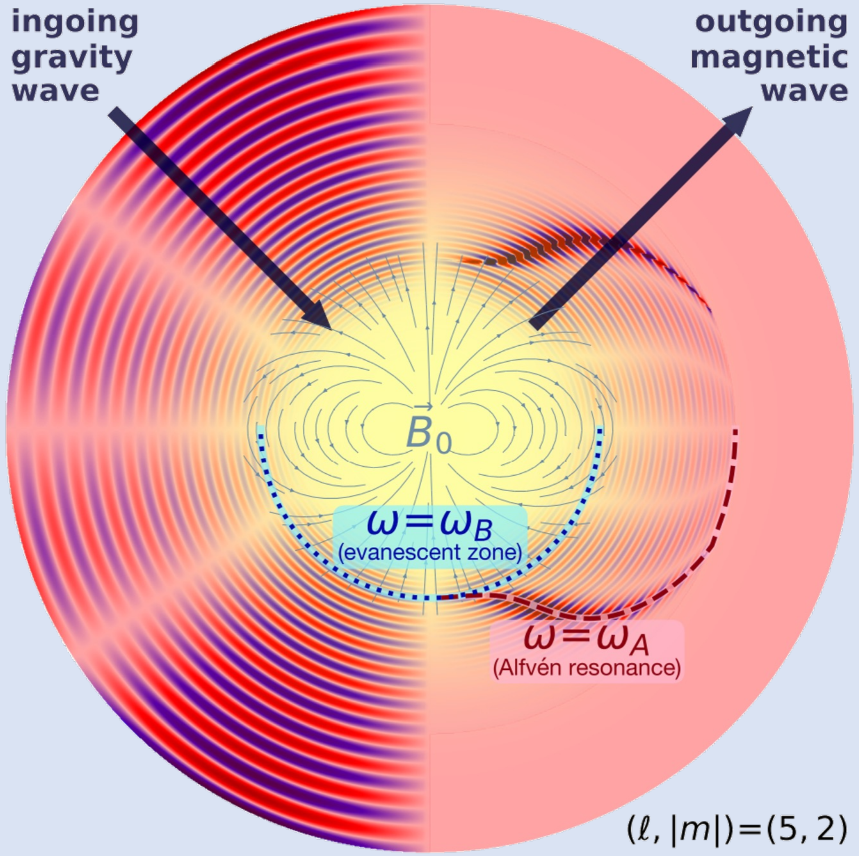
rotation	magnetism
to leading order, effect occurs in the combination	
$\hat{\mathbf{k}} \cdot \boldsymbol{\Omega}$	$\mathbf{k} \cdot \mathbf{B}$
using $k_r \gg k_h$, apply "traditional approximation"	
$\boldsymbol{\Omega} \to \Omega \cos\theta\, \hat{r}$	$\mathbf{B} \to B(r) \cos\theta\, \hat{r}$
horizontal problem solved by "modified spherical harmonics" parameterized by	
$q = 2\Omega/\omega$	$b = k_r v_{Ar}/\omega$
solve radial problem using λ as a function of	
$q = 2\Omega/\omega$	$a = b/\sqrt{\lambda} \sim \omega_B^2/\omega^2$

horizontal structure of strong magnetogravity modes

all modes are refracted to infinite wavenumber at a cutoff height, and are presumably suppressed



tesseral/sectoral modes (m ≠ 0) have sharp fluid features at the critical latitudes, where assumptions may break down

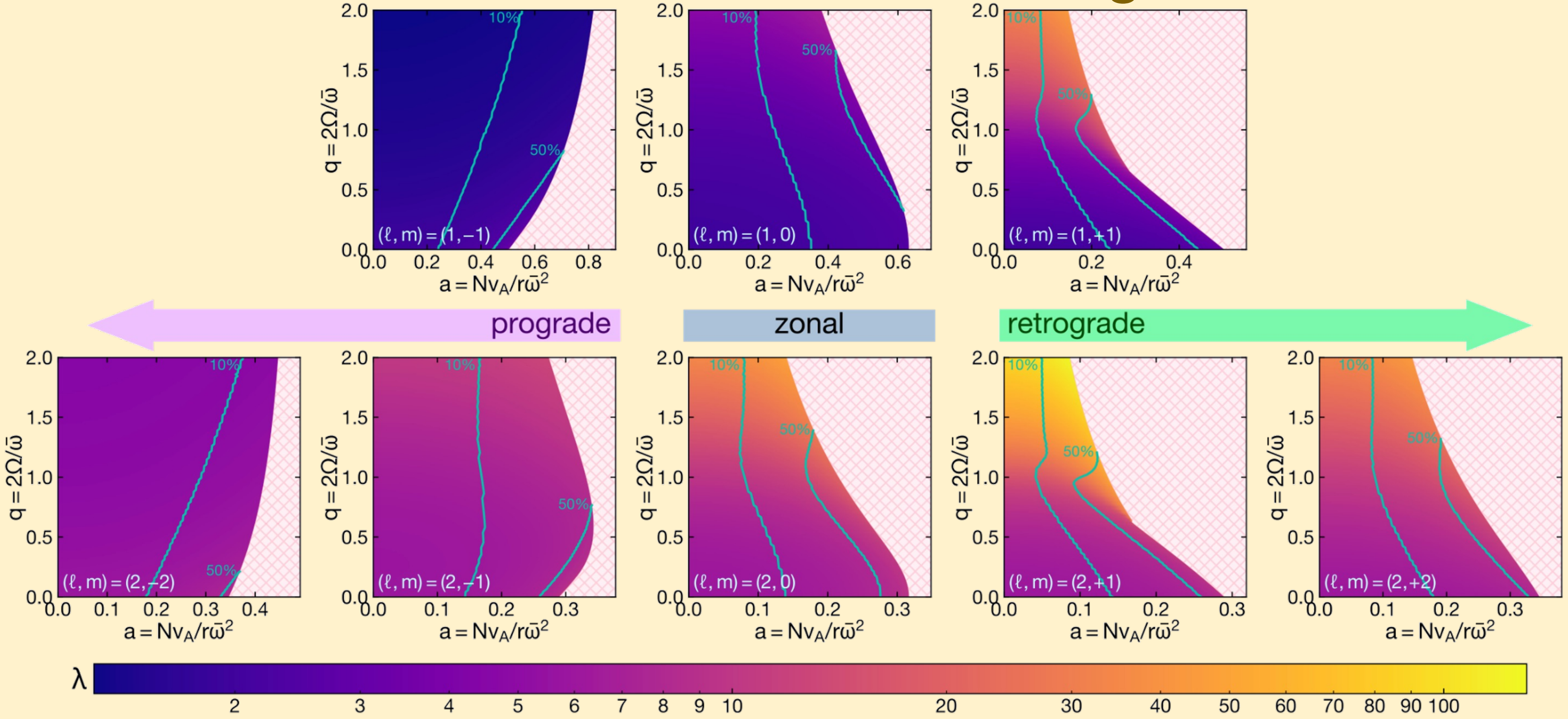


NZR, Fuller, J., *MNRAS* **523**, 1 (2023).

period spacings under rotation *and* magnetism

even before suppression, non-perturbative effects of magnetism may significantly modify oscillation modes

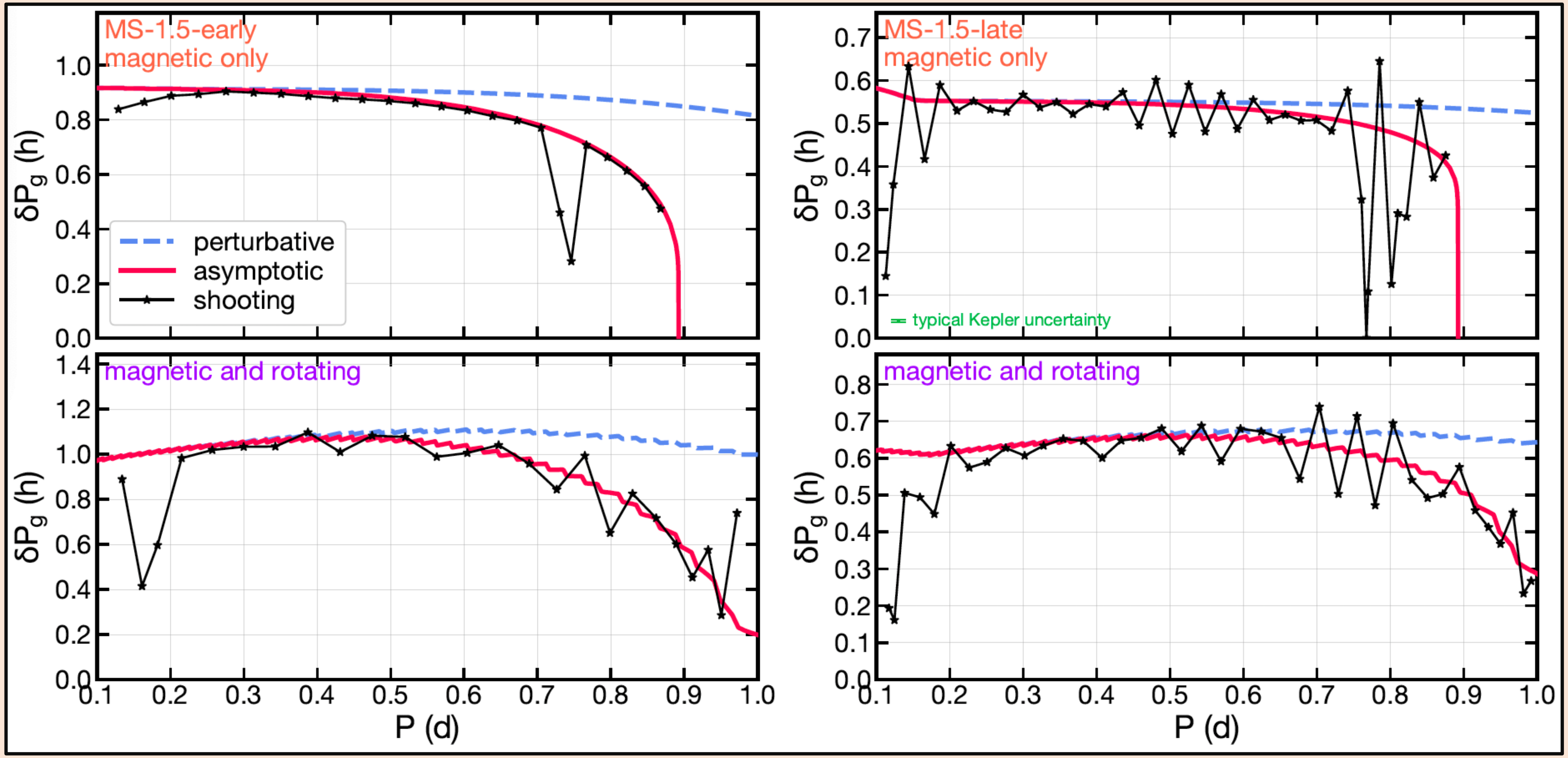
λ under both rotation *and* magnetism



NZR, Ong, J.M.J., Mathis, S., in prep.

period spacings under rotation *and* magnetism

magnetism can introduce sharp drops in the period spacing near mode suppression



NZR, Ong, J.M.J., Mathis, S., in prep.

summary

a "traditional approximation" for magnetism captures the important effects of the radial field on gravity waves

$$\mathbf{k}\cdot\mathbf{B} = k_r B_r$$

$$k_r \to \infty$$

in strong enough fields, magnetogravity waves refract to infinite wavenumbers, where they are efficiently damped

tesseral/sectoral modes develop sharp fluid features and nonzero damping rates due to Alfvén resonances

$$\mathrm{Im}(\omega) \neq 0$$

$$\delta P_g(P) \to 0$$

near suppression, the period spacing may display significant curvature and a sharp fall-off with period

how does any power escape the interior of the star?

what are the non-asymptotic signatures of strong magnetism?

what is the period spacing for suppressed modes?